Theoretical Examination

(T1) **True or False**

Determine if each of the following statements is True or False. In the Summary Answersheet, tick the correct answer (TRUE / FALSE) for each statement. No justifications are necessary for this question.

(T1.1) In a photograph of the clear sky on a Full Moon night with a sufficiently long exposure, the colour of the sky would appear blue as in daytime. **2**

(T1.2) An astronomer at Bhubaneswar marks the position of the Sun on the sky at 05:00 UT every day of the year. If the Earth's axis were perpendicular to its orbital plane, these positions would trace an arc of a great circle. **2**

(T1.3) If the orbital period of a certain minor body around the Sun in the ecliptic plane is less than the orbital period of Uranus, then its orbit must necessarily be fully inside the orbit of Uranus. **2**

(T1.4) The centre of mass of the solar system is inside the Sun at all times. **2**

(T1.5) A photon is moving in free space. As the Universe expands, its momentum decreases. **2**

(T2) **Gases on Titan** **10**

Gas particles in a planetary atmosphere have a wide distribution of speeds. If the r.m.s. (root mean square) thermal speed of particles of a particular gas exceeds $1/6$ of the escape speed, then most of that gas will escape from the planet. What is the minimum atomic weight (relative atomic mass), A_{min}, of an ideal monatomic gas so that it remains in the atmosphere of Titan?

Given, mass of Titan $M_{\mathrm{T}} = 1.23 \times 10^{23}$ kg, radius of Titan $R_{\mathrm{T}} = 2575$ km, surface temperature of Titan $T_{\mathrm{T}} = 93.7$ K.

(T3) **Early Universe**

Cosmological models indicate that radiation energy density, ρ_{r}, in the Universe is proportional to $(1+z)^4$, and the matter energy density, ρ_{m}, is proportional to $(1+z)^3$, where z is the redshift. The dimensionless density parameter, Ω, is given as $\Omega = \rho/\rho_{\mathrm{c}}$, where ρ_{c} is the critical energy density of the Universe. In the present Universe, the density parameters corresponding to radiation and matter, are $\Omega_{\mathrm{r_0}} = 10^{-4}$ and $\Omega_{\mathrm{m_0}} = 0.3$, respectively.

(T3.1) Calculate the redshift, z_{e}, at which radiation and matter energy densities were equal. **3**

(T3.2) Assuming that the radiation from the early Universe has a blackbody spectrum with a temperature of 2.732 K, estimate the temperature, T_{e}, of the radiation at redshift z_{e}. **4**

(T3.3) Estimate the typical photon energy, E_{ν} (in eV), of the radiation as emitted at redshift z_{e}. **3**

(T4) **Shadows** **10**

An observer in the northern hemisphere noticed that the length of the shortest shadow of a 1.000 m vertical stick on a day was 1.732 m. On the same day, the length of the longest shadow of the same vertical stick was measured to be 5.671 m.

Find the latitude, ϕ, of the observer and declination of the Sun, $\delta_{\odot}$, on that day. Assume the Sun to be a point source and ignore atmospheric refraction.

(T5) **GMRT beam transit** **10**

Giant Metrewave Radio Telescope (GMRT), one of the world's largest radio telescopes at metre wavelengths, is located in western India (latitude: $19°6'$ N, longitude: $74°3'$ E). GMRT consists of 30 dish antennas, each with a diameter of 45.0 m. A single dish of GMRT was held fixed with its axis pointing at a zenith angle of $39°42'$ along the northern meridian such that a radio point source would pass along a diameter of the beam, when it is transiting the meridian.

What is the duration T_{transit} for which this source would be within the FWHM (full width at half maximum) of the beam of a single GMRT dish observing at 200 MHz?

Hint: The FWHM size of the beam of a radio dish operating at a given frequency corresponds to the angular resolution of the dish. Assume uniform illumination.

(T6) **Cepheid Pulsations**

The star β-Doradus is a Cepheid variable star with a pulsation period of 9.84 days. We make a simplifying assumption that the star is brightest when it is most contracted (radius being R_1) and it is faintest when it is most expanded (radius being R_2). For simplicity, assume that the star maintains its spherical shape and behaves as a perfect black body at every instant during the entire cycle. The bolometric magnitude of the star varies from 3.46 to 4.08. From Doppler measurements, we know that during pulsation the stellar surface expands or contracts at an average radial speed of 12.8 km s^{-1}. Over the period of pulsation, the peak of thermal radiation (intrinsic) of the star varies from 531.0 nm to 649.1 nm.

(T6.1) Find the ratio of radii of the star in its most contracted and most expanded states (R_1/R_2). **7**

(T6.2) Find the radii of the star (in metres) in its most contracted and most expanded states (R_1 and R_2). **3**

(T6.3) Calculate the flux of the star, F_2, when it is in its most expanded state. **5**

(T6.4) Find the distance to the star, D_{star}, in parsecs. **5**

(T7) **Telescope optics**

In a particular ideal refracting telescope of focal ratio $f/5$, the focal length of the objective lens is 100 cm and that of the eyepiece is 1 cm.

(T7.1) What is the angular magnification, m_0, of the telescope? What is the length of the telescope, L_0, i.e. the distance between its objective and eyepiece? **4**

An introduction of a concave lens (Barlow lens) between the objective lens and the prime focus is a common way to increase the magnification without a large increase in the length of the telescope. A Barlow lens of focal length 1 cm is now introduced between the objective and the eyepiece to double the magnification.

(T7.2) At what distance, d_{B}, from the prime focus must the Barlow lens be kept in order to obtain this desired double magnification? **6**

(T7.3) What is the increase, ΔL, in the length of the telescope? **4**

A telescope is now constructed with the same objective lens and a CCD detector placed at the prime focus (without any Barlow lens or eyepiece). The size of each pixel of the CCD detector is 10 μm.

(T7.4) What will be the distance in pixels between the centroids of the images of the two stars , n_{p}, on the CCD, if they are $20''$ apart on the sky? **6**

(T8) **U-Band photometry**

A star has an apparent magnitude $m_{\mathrm{U}} = 15.0$ in the *U*-band. The *U*-band filter is ideal, i.e., it has perfect (100%) transmission within the band and is completely opaque (0% transmission) outside the band. The filter is centered at 360 nm, and has a width of 80 nm. It is assumed that the star also has a flat energy spectrum with respect to frequency. The conversion between magnitude, m, in any band and flux density, f, of a star in Jansky ($1\ \mathrm{Jy} = 1 \times 10^{-26}\ \mathrm{W\ Hz^{-1}\ m^{-2}}$) is given by

$$f = 3631 \times 10^{-0.4m}\mathrm{Jy}$$

(T8.1) Approximately how many *U*-band photons, N_0, from this star will be incident normally on a $1\ \mathrm{m}^2$ area at the top of the Earth's atmosphere every second? **8**

This star is being observed in the *U*-band using a ground based telescope, whose primary mirror has a diameter of 2.0 m. Atmospheric extinction in *U*-band during the observation is 50%. You may assume that the seeing is diffraction limited. Average surface brightness of night sky in *U*-band was measured to be $22.0\ \mathrm{mag/arcsec^2}$.

(T8.2) What is the ratio, R, of number of photons received per second from the star to that received from the sky, when measured over a circular aperture of diameter $2''$? **8**

(T8.3) In practice, only 20% of U-band photons falling on the primary mirror are detected. How many photons, N_{t}, from the star are detected per second? **4**

(T9) **Mars Orbiter Mission**

India's Mars Orbiter Mission (MOM) was launched using the Polar Satellite Launch Vehicle (PSLV) on 5 November 2013. The dry mass of MOM (body + instruments) was 500 kg and it carried fuel of mass 852 kg. It was initially placed in an elliptical orbit around the Earth with perigee at a height of 264.1 km and apogee at a height of 23903.6 km, above the surface of the Earth. After raising the orbit six times, MOM was transferred to a trans-Mars injection orbit (Hohmann orbit).

The first such orbit-raising was performed by firing the engines for a very short time near the perigee. The engines were fired to change the orbit without changing the plane of the orbit and without changing its perigee. This gave a net impulse of 1.73×10^5 kg m s^{-1} to the satellite. Ignore the change in mass due to burning of fuel.

(T9.1) What is the height of the new apogee, h_a above the surface of the Earth, after this engine burn? **14**

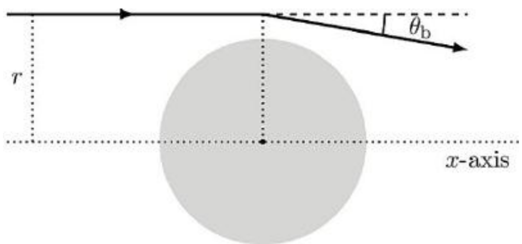
(T9.2) Find the eccentricity (e) of the new orbit after the burn and the new orbital period (P) of MOM in hours. **6**

(T10) **Gravitational Lensing Telescope**

Einstein's General Theory of Relativity predicts bending of light around massive bodies. For simplicity, we assume that the bending of light happens at a single point for each light ray, as shown in the figure. The angle of bending, θ_b, is given by

$$\theta_\mathrm{b} = \frac{2R_\mathrm{sch}}{r}$$

where R_sch is the Schwarzschild radius associated with that gravitational body. We call r, the distance of the incoming light ray from the parallel x-axis passing through the centre of the body, as the "impact parameter".



A massive body thus behaves somewhat like a focusing lens. The light rays coming from infinite distance beyond a massive body, and having the same impact parameter r, converge at a point along the axis, at a distance f_r from the centre of the massive body. An observer at that point will benefit from huge amplification due to this gravitational focusing. The massive body in this case is being used as a Gravitational Lensing Telescope for amplification of distant signals.

(T10.1) Consider the possibility of our Sun as a gravitational lensing telescope. Calculate the shortest distance, f_min, from the centre of the Sun (in A. U.) at which the light rays can get focused. **6**

(T10.2) Consider a small circular detector of radius a, kept at a distance f_min centered on the x-axis and perpendicular to it. Note that only the light rays which pass within a certain annulus (ring) of width h (where $h \ll R_\odot$) around the Sun would encounter the detector. The amplification factor at the detector is defined as the ratio of the intensity of the light incident on the detector in the presence of the Sun and the intensity in the absence of the Sun. **8**

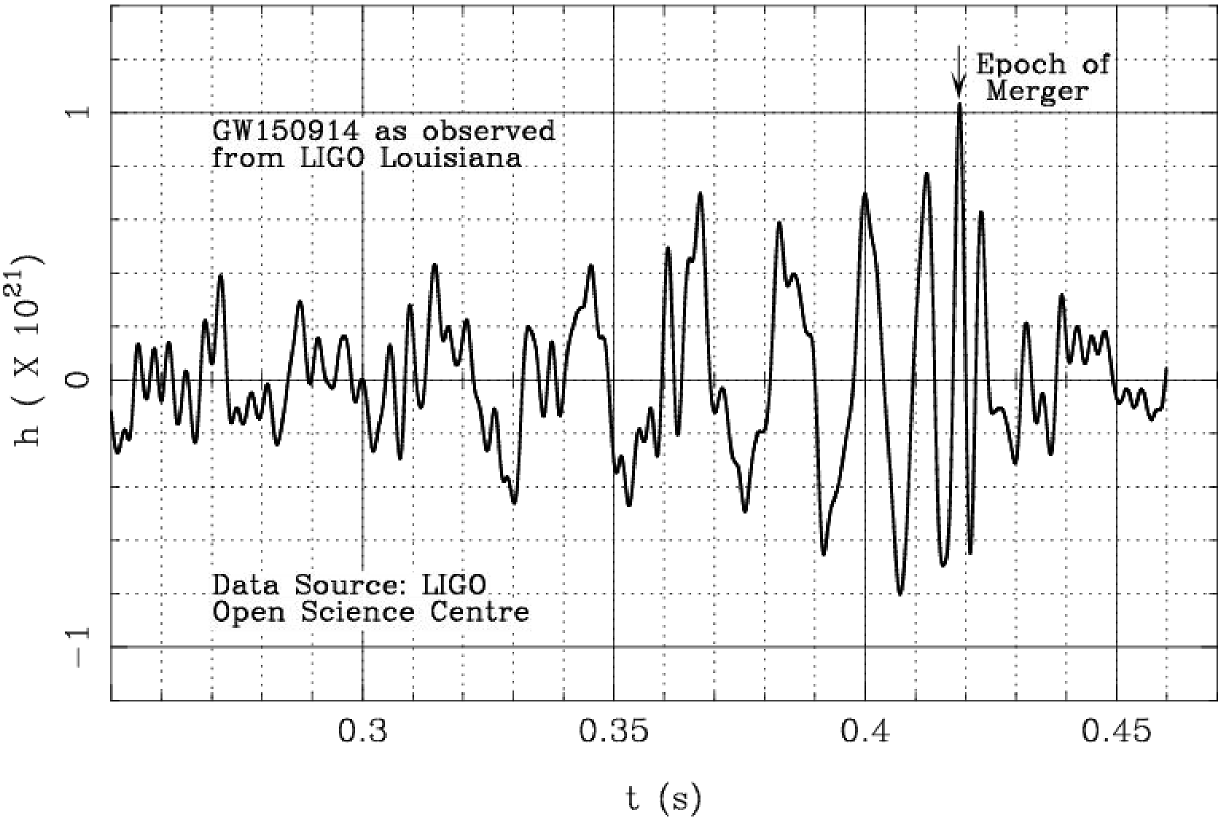
Express the amplification factor, A_m, at the detector in terms of $R_\odot$ and a.

(T10.3) Consider a spherical mass distribution, such as dark matter in a galaxy cluster, through which light rays can pass while undergoing gravitational bending. Assume for simplicity that for the gravitational bending with impact parameter, r, only the mass $M(r)$ enclosed inside the radius r is relevant. **6**

What should be the mass distribution, $M(r)$, such that the gravitational lens behaves like an ideal optical convex lens?

(T11) **Gravitational Waves**

The first signal of gravitational waves was observed by two advanced LIGO detectors at Hanford and Livingston, USA in September 2015. One of these measurements (strain vs time in seconds) is shown in the accompanying figure. In this problem, we will interpret this signal in terms of a small test mass m orbiting around a large mass M (i.e., $m \ll M$), by considering several models for the nature of the central mass.



The test mass loses energy due to the emission of gravitational waves. As a result the orbit keeps on shrinking, until the test mass reaches the surface of the object, or in the case of a black hole, the innermost stable circular orbit – ISCO – which is given by $R_{\text{ISCO}} = 3R_{\text{sch}}$, where R_{sch} is the Schwarzschild radius of the black hole. This is the "epoch of merger". At this point, the amplitude of the gravitational wave is maximum, and so is its frequency, which is always twice the orbital frequency. In this problem, we will only focus on the gravitational waves before the merger, when Kepler's laws are assumed to be valid. After the merger, the form of gravitational waves will drastically change.

(T11.1) Consider the observed gravitational waves shown in the figure above. Estimate the time period, T_0, and hence calculate the frequency, f_0, of gravitational waves just before the epoch of merger. **3**

(T11.2) For any main sequence (MS) star, the radius of the star, R_{MS}, and its mass, M_{MS}, are related by a power law given as, **10**

$$R_{\text{MS}} \propto (M_{\text{MS}})^{\alpha}$$

$$\text{where } \alpha = 0.8 \quad \text{for } M_{\odot} < M_{\text{MS}}$$

$$= 1.0 \quad \text{for } 0.08M_{\odot} \leq M_{\text{MS}} \leq M_{\odot}$$

If the central object were a main sequence star, write an expression for the maximum frequency of gravitational waves, f_{MS}, in terms of mass of the star in units of solar masses $(M_{\text{MS}}/M_{\odot})$ and α.

(T11.3) Using the above result, determine the appropriate value of α that will give the maximum possible frequency of gravitational waves, $f_{\text{MS,max}}$ for any main sequence star. Evaluate this frequency. **9**

(T11.4) White dwarf (WD) stars have a maximum mass of 1.44 $M_{\odot}$ (known as the Chandrasekhar limit) and obey the mass-radius relation $R \propto M^{-1/3}$. The radius of a solar mass white dwarf is equal **8** to 6000 km. Find the highest frequency of emitted gravitational waves, $f_{\mathrm{WD,max}}$, if the test mass is orbiting a white dwarf.

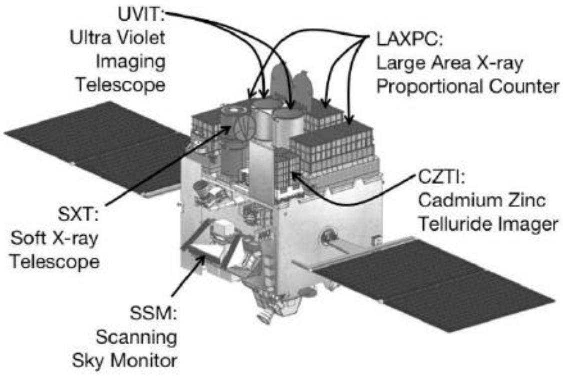
(T11.5) Neutron stars (NS) are a peculiar type of compact objects which have masses between 1 and $3M_{\odot}$ and radii in the range $10 - 15$ km. Find the range of frequencies of emitted gravitational waves, $f_{\mathrm{NS,min}}$ and $f_{\mathrm{NS,max}}$, if the test mass is orbiting a neutron star at a distance close to the neutron star radius. **8**

(T11.6) If the test mass is orbiting a black hole (BH), write the expression for the frequency of emitted gravitational waves, f_{BH}, in terms of mass of the black hole, M_{BH}, and the solar mass $M_{\odot}$. **7**

(T11.7) Based only on the time period (or frequency) of gravitational waves before the epoch of merger, determine whether the central object can be a main sequence star (MS), a white dwarf (WD), a neutron star (NS), or a black hole (BH). Tick the correct option in the Summary Answersheet. Estimate the mass of this object, M_{obj}, in units of $M_{\odot}$. **5**

(T12) **AstroSat**

India astronomy satellite, AstroSat, launched in September 2015, has five different instruments.



In this question, we will discuss three of these instruments (SXT, LAXPC, CZTI), which point in the same direction and observe in X-ray wavelengths. The details of these instruments are given in the table below.

Instrument	Band [keV]	Collecting Area [m²]	Effective Photon Detection Efficiency	Saturation level [counts]	No. of Pixels
SXT	0.3 – 80	0.067	60%	15000 (total)	512 x 512
LAXPC	3 – 80	1.5	40%	50000 (in any one counter) or 200000 (total)	---
CZTI	10 – 150	0.09	50%	---	4 x 4096

You should note that LAXPC energy range is divided into 8 different energy band counters of equal bandwidth with no overlap.

(T12.1) Some X-ray sources like Cas A have a prominent emission line at 0.01825 nm corresponding to a radioactive transition of ^{44}Ti. Suppose there exists a source which emits only one bright emission line corresponding to this transition. What should be the minimum relative velocity (v) of the source, which will make the observed peak of this line to get registered in a different energy band counter of LAXPC as compared to a source at rest? **13**

These instruments were used to observe an X-ray source (assumed to be a point source), whose energy spectrum followed the power law,

$$F(E) = KE^{-2/3} \qquad \text{[in units of counts/keV/m}^2\text{/s]}$$

where E is the energy in keV, K is a constant and $F(E)$ is photon flux density at that energy. Photon flux density, by definition, is given for per unit collecting area (m^2) per unit bandwidth (keV) and per unit time (seconds). From prior observations, we know that the source has a flux density of 10 counts/keV/m^2/s at 1 keV, when measured by a detector with 100% photon detection efficiency. The “counts” here mean the number of photons reported by the detector.

As the source flux follows the power law given above, we know that for a given energy range from E_1 (lower energy) to E_2 (higher energy) the total photon flux (F_T) will be given by

$$F_T = 3K\ (E_2^{1/3} - E_1^{1/3}) \qquad \text{[in units of counts/m}^2\text{/s]}$$

(T12.2) Estimate the incident flux density from the source at 1 keV, 5 keV, 40 keV and 100 keV. Also estimate what will be the total count per unit bandwidth recorded by each of the instruments at these energies for an exposure time of 200 seconds. **8**

(T12.3) For this source, calculate the maximum exposure time (t_S), without suffering from saturation, for the CCD of SXT. **4**

(T12.4) If the source became 3500 times brighter, calculate the expected counts per second in LAXPC counter 1, counter 8 as well as total counts across the entire energy range. If we observe for longer period, will the counter saturate due to any individual counter or due to the total count? Tick the appropriate box in the Summary Answersheet. **8**

(T12.5) Assume that the counts reported by CZTI due to random fluctuations in electronics are about 0.00014 counts per pixel per keV per second at all energy levels. Any source is considered as “detected” when the SNR (signal to noise ratio) is at least 3. What is minimum exposure time, t_c, needed for the source above to be detected in CZTI? **10**
Note that the “noise” in a detector is equal to the square root of the counts due to random fluctuations.

(T12.6) Let us consider the situation where the source shows variability in number flux, so that the factor K increases by 20%. AstroSat observed this source for 1 second before the change and 1 second after this change in brightness. Calculate the counts measured by SXT, LAXPC and CZTI in both the observations. Which instrument is best suited to detect this change? Tick the appropriate box in the Summary Answersheet. **7**

(D1) **Binary Pulsar**

Through systematic searches during the past decades, astronomers have found a large number of millisecond pulsars (spin period < 10 ms). Majority of these pulsars are found in binaries, with nearly circular orbits.

For a pulsar in a binary orbit, the measured pulsar spin period (P) and the measured line-of-sight acceleration (a) both vary systematically due to orbital motion. For circular orbits, this variation can be described mathematically in terms of orbital phase ϕ $(0 \le \phi \le 2\pi)$ as,

$$P(\phi) = P_0 + P_{\mathrm{t}}\, cos\phi \qquad \text{where } P_t = \frac{2\pi P_0 r}{cP_{\mathrm{B}}}$$

$$a(\phi) = -a_{\mathrm{t}}\, sin\phi \qquad \text{where } a_{\mathrm{t}} = \frac{4\pi^2 r}{P_{\mathrm{B}}^2}$$

where P_{B} is the orbital period of the binary, P_0 is the intrinsic spin period of the pulsar and r is the radius of the orbit.

The following table gives one such set of measurements of P and a at different heliocentric epochs, T, expressed in truncated Modified Julian Days (tMJD), i.e. number of days since MJD = 2,440,000.

No.	T (tMJD)	P (μs)	a (m s^{-2})
1	5740.654	7587.8889	- 0.92 ± 0.08
2	5740.703	7587.8334	- 0.24 ± 0.08
3	5746.100	7588.4100	- 1.68 ± 0.04
4	5746.675	7588.5810	+ 1.67 ± 0.06
5	5981.811	7587.8836	+ 0.72 ± 0.06
6	5983.932	7587.8552	- 0.44 ± 0.08
7	6005.893	7589.1029	+ 0.52 ± 0.08
8	6040.857	7589.1350	+ 0.00 ± 0.04
9	6335.904	7589.1358	+ 0.00 ± 0.02

By plotting $a(\phi)$ as a function of $P(\phi)$, we can obtain a parametric curve. As evident from the relations above, this curve in the period-acceleration plane is an ellipse.

In this problem, we estimate the intrinsic spin period, P_0, the orbital period, P_{B}, and the orbital radius, r, by an analysis of this data set, assuming a circular orbit.

(D1.1) Plot the data, including error bars, in the period-acceleration plane (mark your graph as “D1.1”). **7**

(D1.2) Draw an ellipse that appears to be a best fit to the data (on the same graph “D1.1”). **2**

(D1.3) From the plot, estimate P_0, P_{t} and a_{t}, including error margins. **7**

(D1.4) Write expressions for P_{B} and r in terms of P_0, P_{t}, a_{t}. **4**

(D1.5) Calculate approximate value of P_{B} and r based on your estimations made in (D1.3), including error margins. **6**

(D1.6) Calculate orbital phase, ϕ, corresponding to the epochs of the following five observations in the above table: data rows 1, 4, 6, 8, 9. **4**

(D1.7) Refine the estimate of the orbital period, P_{B}, using the results in part (D1.6) in the following way:

(D1.7a) First determine the initial epoch, T_0, which corresponds to the nearest epoch of zero orbital phase before the first observation. **2**

(D1.7b) The expected time, T_{calc}, of the estimated orbital phase angle of each observation is given by, **7**

$$T_{\mathrm{calc}} = T_0 + \left(n + \frac{\phi}{360°}\right) P_{\mathrm{B}},$$

where n is the number of full cycle of orbital phases that may have elapsed between T_0 and T (or T_{calc}). Estimate n and T_{calc} for each of the five observations in part (D1.6). Note down difference $T_{\mathrm{O-C}}$ between observed T and T_{calc}. Enter these calculations in the table given in the Summary Answersheet.

(D1.7c) Plot $T_{\mathrm{O-C}}$ against n (mark your graph as "D1.7"). **4**

(D1.7d) Determine the refined values of the initial epoch, $T_{0,\mathrm{r}}$, and the orbital period, $P_{\mathrm{B,r}}$. **7**

(D2) **Distance to the Moon**

Geocentric ephemerides of the Moon for September 2015 are given in the form of a table. Each reading was taken at 00:00 UT.

Date	R.A. (α)			Dec. (δ)			Angular Size (θ)	Phase (ϕ)	Elongation
	h	m	s	°	'	"	"		Of Moon
Sep 01	0	36	46.02	3	6	16.8	1991.2	0.927	148.6° W
Sep 02	1	33	51.34	7	32	26.1	1974.0	0.852	134.7° W
Sep 03	2	30	45.03	11	25	31.1	1950.7	0.759	121.1° W
Sep 04	3	27	28.48	14	32	4.3	1923.9	0.655	107.9° W
Sep 05	4	23	52.28	16	43	18.2	1896.3	0.546	95.2° W
Sep 06	5	19	37.25	17	55	4.4	1869.8	0.438	82.8° W
Sep 07	6	14	19.23	18	7	26.6	1845.5	0.336	70.7° W
Sep 08	7	7	35.58	17	23	55.6	1824.3	0.243	59.0° W
Sep 09	7	59	11.04	15	50	33.0	1806.5	0.163	47.5° W
Sep 10	8	49	0.93	13	34	55.6	1792.0	0.097	36.2° W
Sep 11	9	37	11.42	10	45	27.7	1780.6	0.047	25.1° W
Sep 12	10	23	57.77	7	30	47.7	1772.2	0.015	14.1° W
Sep 13	11	9	41.86	3	59	28.8	1766.5	0.001	3.3° W
Sep 14	11	54	49.80	0	19	50.2	1763.7	0.005	7.8° E
Sep 15	12	39	50.01	-3	20	3.7	1763.8	0.026	18.6° E
Sep 16	13	25	11.64	-6	52	18.8	1767.0	0.065	29.5° E
Sep 17	14	11	23.13	-10	9	4.4	1773.8	0.120	40.4° E
Sep 18	14	58	50.47	-13	2	24.7	1784.6	0.189	51.4° E
Sep 19	15	47	54.94	-15	24	14.6	1799.6	0.270	62.5° E
Sep 20	16	38	50.31	-17	6	22.8	1819.1	0.363	73.9° E
Sep 21	17	31	40.04	-18	0	52.3	1843.0	0.463	85.6° E
Sep 22	18	26	15.63	-18	0	41.7	1870.6	0.567	97.6° E
Sep 23	19	22	17.51	-17	0	50.6	1900.9	0.672	110.0° E
Sep 24	20	19	19.45	-14	59	38.0	1931.9	0.772	122.8° E
Sep 25	21	16	55.43	-11	59	59.6	1961.1	0.861	136.2° E
Sep 26	22	14	46.33	-8	10	18.3	1985.5	0.933	150.0° E
Sep 27	23	12	43.63	-3	44	28.7	2002.0	0.981	164.0° E
Sep 28	0	10	48.32	0	58	58.2	2008.3	1.000	178.3° E
Sep 29	1	9	5.89	5	38	54.3	2003.6	0.988	167.4° W
Sep 30	2	7	39.02	9	54	16.1	1988.4	0.947	153.2° W

The composite graphic[1] below shows multiple snapshots of the Moon taken at different times during the total lunar eclipse, which occurred in this month. For each shot, the centre of frame was coinciding with the central north-south line of umbra.

For this problem, assume that the observer is at the centre of the Earth and angular size refers to angular diameter of the object / shadow.

[1] Credit: NASA's Scientific Visualization Studio

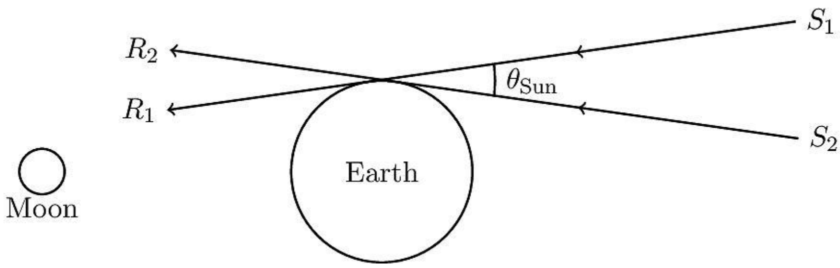
(D2.1) In September 2015, apogee of the lunar orbit is closest to **3**
New Moon / First Quarter / Full Moon / Third Quarter.
Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary.

(D2.2) In September 2015, the ascending node of lunar orbit with respect to the ecliptic is closest to **4**
New Moon / First Quarter / Full Moon / Third Quarter.
Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary.

(D2.3) Estimate the eccentricity, e, of the lunar orbit from the given data. **4**

(D2.4) Estimate the angular size of the umbra, θ_{umbra}, in terms of the angular size of the Moon, θ_{Moon}. Show your working on the image given on the backside of the Summary Answersheet. **8**

(D2.5) The angle subtended by the Sun at Earth on the day of the lunar eclipse is known to be $\theta_{\text{Sun}} = 1915.0''$. In the figure below, S_1R_1 and S_2R_2 are rays coming from diametrically opposite ends of the solar disk. The figure is not to scale. **9**



Calculate the angular size of the penumbra, θ_{penumbra}, in terms of θ_{Moon}. Assume the observer to be at the centre of the Earth.

(D2.6) Let θ_{Earth} be angular size of the Earth as seen from the centre of the Moon. Calculate the angular size of the Moon, θ_{Moon}, as would be seen from the centre of the Earth on the eclipse day in terms of θ_{Earth}. **5**

(D2.7) Estimate the radius of the Moon, R_{Moon}, in km from the results above. **3**

(D2.8) Estimate the shortest distance, r_{perigee}, and the farthest distance, r_{apogee}, to the Moon. **4**

(D2.9) Use appropriate data from September 10 to estimate the distance, d_{Sun}, to the Sun from the Earth. **10**

(D3) **Type IA Supernovae**

Supernovae of type Ia are considered very important for the measurements of large extragalactic distances. The brightening and subsequent dimming of these explosions follow a characteristic light curve, which helps in identifying these as supernovae of type Ia.

Light curves of all type Ia supernovae can be fit to the same model light curve, when they are scaled appropriately. In order to achieve this, we first have to express the light curves in the reference frame of the host galaxy by taking care of the cosmological stretching/dilation of all observed time intervals, Δt_{obs}, by a factor of $(1 + z)$. The time interval in the rest frame of the host galaxy is denoted by Δt_{gal}.

The rest frame light curve of a supernova changes by two magnitudes compared to the peak in a time interval Δt_0 after the peak. If we further scale the time intervals by a factor of s (i.e. $\Delta t_s = s\Delta t_{\text{gal}}$) such that the scaled value of Δt_0 is the same for all supernovae, the light curves turn out to have the same shape. It also turns out that s is related linearly to the absolute magnitude, M_{peak}, at the peak luminosity for the supernova. That is, we can write

$$s = a + bM_{\text{peak}} ,$$

where a and b are constants. Knowing the scaling factor, one can determine absolute magnitudes of supernovae at unknown distances from the above linear equation.

The table below contains data for three supernovae, including their distance moduli, μ (for the first two), their recession speed, cz, and their apparent magnitudes, m_{obs}, at different times. The time $\Delta t_{\text{obs}} \equiv t - t_{\text{peak}}$ shows number of days from the date at which the respective supernova reached peak brightness. The observed magnitudes have already been corrected for interstellar as well as atmospheric extinction.

Name	**SN2006TD**	**SN2006IS**	**SN2005LZ**
μ (mag)	**34.27**	**35.64**	
cz (km s^{-1})	**4515**	**9426**	**12060**
Δt_{obs} (days)	m_{obs} (mag)	m_{obs} (mag)	m_{obs} (mag)
-15.00	19.41	18.35	20.18
-10.00	17.48	17.26	18.79
-5.00	16.12	16.42	17.85
0.00	15.74	16.17	17.58
5.00	16.06	16.41	17.72
10.00	16.72	16.82	18.24
15.00	17.53	17.37	18.98
20.00	18.08	17.91	19.62
25.00	18.43	18.39	20.16
30.00	18.64	18.73	20.48

(D3.1) Compute Δt_{gal} values for all three supernovae, and fill them in the given blank boxes in the data tables on the BACK side of the Summary Answersheet. On a graph paper, plot the points and draw the three light curves in the rest frame (mark your graph as "D3.1"). **15**

(D3.2) Take the scaling factor, s_2, for the supernova SN2006IS to be 1.00. Calculate the scaling factors, s_1 and s_3, for the other two supernovae SN2006TD and SN2005LZ, respectively, by calculating Δt_0 for them. **5**

(D3.3) Compute the scaled time differences, Δt_s, for all three supernovae. Write the values for Δt_s in the same data tables on the Summary Answersheet. On another graph paper, plot the points and draw the 3 light curves to verify that they now have an identical profile (mark your graph as "D3.3"). **14**

(D3.4) Calculate the absolute magnitudes at peak brightness, $M_{\text{peak},1}$, for SN2006TD and $M_{\text{peak},2}$, for SN2006IS. Use these values to calculate a and b. **6**

(D3.5) Calculate the absolute magnitude at peak brightness, $M_{\text{peak},3}$, and distance modulus, μ_3, for SN2005LZ. **4**

(D3.6) Use the distance modulus μ_3 to estimate the value of Hubble's constant, H_0. Further, estimate the characteristic age of the universe, T_{H}. **6**

Observational Examination (OP)

(OP1) Eight well known historical supernovae will appear in the projected sky one at a time (not necessarily in chronological order). You have to identify the appropriate map (Map 1 / Map 2) where a particular supernova belongs and mark it in the corresponding map with '+' sign and write codes 'S1' to 'S8' besides it. **40**

Each supernova code will be projected on dome for 10 seconds, followed by appearance of supernova for 60 seconds and then 20 seconds for you to mark the answers.

(OP1.1) For S1, S2, S3, S4 and S5, the projected sky corresponds to the sky as seen from Rio de Janeiro on the midnight of 21^{st} May.

(OP1.2) For S6, S7 and S8, the projected sky corresponds to the sky as seen from Beijing on the midnight of 20^{th} November. There will be a gap of two minute after S5 for change over and adaptation to new sky.

(OP2) We are now projecting sky of another planet. The sky will be slowly rotated for 5 minutes . Identify the visible celestial pole of this planet and mark it with a '+' sign and label it as 'P' on the appropriate map (Map 1 / Map 2). **10**

Observational Examination (OM)

(OM1) Mark any 5 (five) of the following stars on the map by putting a circle (O) around the appropriate star and writing its code next to it. If you mark more than 5 stars, only the first 5 in serial order will be considered. **20**

Code	Name	Bayer Name
S1	Caph	β Cas
S2	Asellus Australis	δ Cnc
S3	Acrux	α Cru
S4	Alphard	α Hya

Code	Name	Bayer Name
S5	Sheliak	β Lyr
S6	Albireo	β Cyg
S7	Rasalhague	α Oph
S8	Kaus Australis	ϵ Sgr

(OM2) Mark location of any 3 (three) of the following galaxies on the map by putting a '+' sign at appropriate place in the map and writing its code next to it. If you mark more than 3 galaxies, only the first 3 in serial order will be considered. **15**

Code	Name	M number
G1	Triangulum Galaxy	M 33
G2	Whirlpool Galaxy	M 51
G3	Southern Pinwheel Galaxy	M 83

Code	Name	M number
G4	Virgo A	M 87
G5	Sombrero Galaxy	M 104

(OM3) Draw ecliptic on the map and label it as 'E'. **5**

(OM4) Show position of Autumnal Equinox (descending node of the ecliptic) on the map by a '+' sign and label it as 'A'. **5**

(OM5) Draw local meridian for Bhubaneswar on Winter Solstice day (22nd December) at local midnight and label it as 'M'. **5**

Observational Examination (OT)

When you arrive at your observing station, **DO NOT** disturb the telescope before attempting the first question (OT1).

(OT1) The telescope is already set to a deep sky object. Identify the object and tick the correct box in the Summary Answersheet. **10**

Note: You can use any technique to identify the object. However, if you disturb the telescope, you will **NOT** be helped to bring it back to the original position.

(OT2)

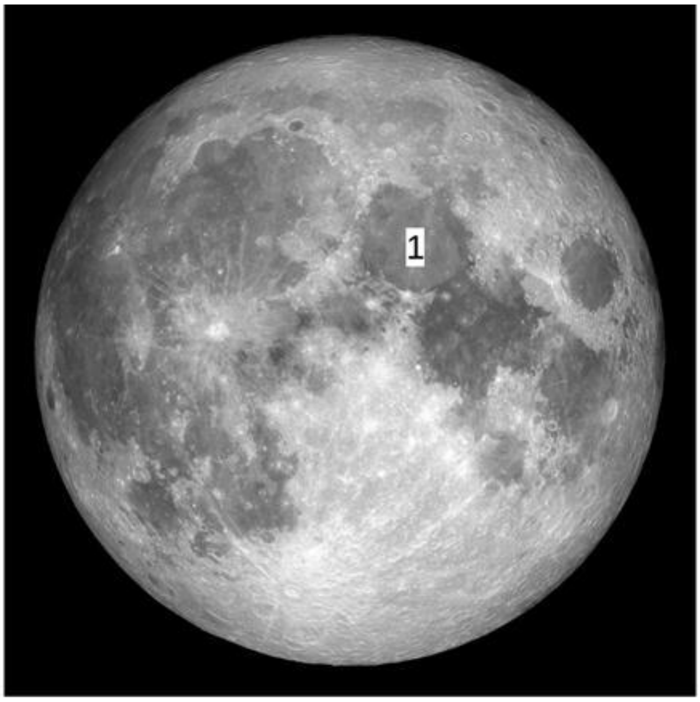
(OT2.1) Point the telescope to M45. Show the object to the examiner. **5**

Note: 1. After 5 minutes, 1 mark will be deducted for a delay of every minute (or part thereof) in pointing the telescope.

2. You have a single chance to be evaluated. If your pointing is incorrect the examiner will change the pointing to M45 for the next part of the question.

(OT2.2) Your Summary Answersheet shows telescopic field of M45. In the image, seven (7) brightest stars of the cluster are replaced by '+' sign. Compare the image with the field you see in the telescope and number the '+' marks from 1 to 7 in the order of decreasing brightness (brightest is 1 and faintest is 7) of the corresponding stars. **15**

(OT3) The examiner will give you a moon filter, an eyepiece with a cross-wire and a stopwatch. Point the telescope towards the Moon. Attach the filter to the telescope. On the surface of the Moon, you will see several "seas" (maria) which are nearly circular in shape. Estimate the diameter of Mare Serenitatis, D_{MSr}, labelled as "1" in the figure below, as a fraction of the lunar diameter, D_{Moon}, by measuring the telescope drift times, t_{Moon} and t_{MSr}, for the Moon and the mare, respectively. **20**



Group Examination

(G1) A spacecraft of mass m and velocity $\vec{v}$ approaches a massive planet of mass M and orbital velocity $\vec{u}$, as measured by an inertial observer. We consider a special case, where the incoming trajectory of the spacecraft is designed in a way such that velocity vector of the planet does not change direction due to the gravitational boost given to the spacecraft. In this case, the gravitational boost to the velocity the spacecraft can be estimated using conservation laws by measuring asymptotic velocity of the spacecraft before and after the interaction and angle of approach of the spacecraft.

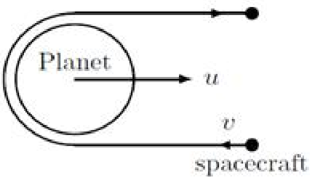


Figure 1

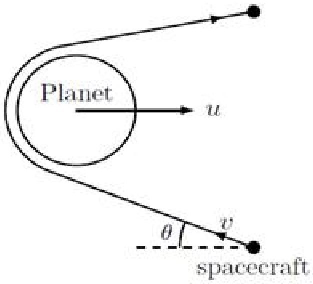


Figure 2

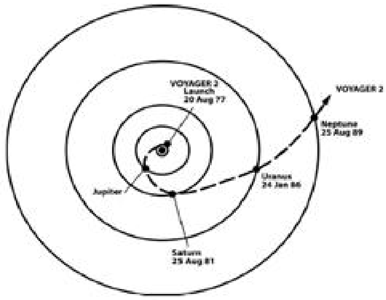


Figure 3

(G1.1) What will be the final velocity ($\vec{v_f}$) of the spacecraft, if $\vec{v}$ and $\vec{u}$ are exactly anti-parallel (see Figure 1). **3**

(G1.2) Simplify the expression for the case where $m << M$. **1**

(G1.3) If angle between $\vec{v}$ and $-\vec{u}$ is θ and $m << M$ (see Figure 2), use results above to write expression for the magnitude of final velocity (v_f). **3**

(G1.4) Table on the last page gives data of Voyager-2 spacecraft for a few months in the year 1979 as it passed close to Jupiter. Assume that the observer is located at the centre of the Sun. The distance from the observer is given in AU and λ is heliocentric ecliptic longitude in degrees. Assume all objects to be in the ecliptic plane. Assume that the orbit of the Earth to be circular. Plot appropriate column against the date of observation to find the date at which the spacecraft was closest to the Jupiter, and label the graph as G1.4. **8**

(G1.5) Find the Earth-Jupiter distance, (d_{E-J}) on the day of the encounter. **4**

(G1.6) On the day of the encounter, around what standard time (t_{std}) had the Jupiter transited the meridian in the sky of Bhubaneswar ($20.27°$ N; $85.84°$E; UT $+$ 05:30)? **6**

(G1.7) Speed of the spacecraft (in km s^{-1}) as measured by the same observer on some dates before the encounter and some dates after the encounter are given below. Here day n is the date of encounter. Use these data to find the orbital speed of Jupiter (u) on the date of encounter and angle θ. **12**

date	n-45	n-35	n-25	n-15	n-5	n
v_{tot}	10.1408	10.0187	9.9078	9.8389	10.2516	25.5150
date	n+5	n+15	n+25	n+35	n+45	
v_{tot}	21.8636	21.7022	21.5580	21.3812	21.2365	

(G1.8) Find eccentricity, e_J, of Jupiter's orbit. **8**

(G1.9) Find heliocentric ecliptic longitude, λ_p, of Jupiter's perihelion point. **5**

Month	Date	λ (∘)	Distance (AU)
June	1	135.8870	5.1589731906
June	2	135.9339	5.1629499712
June	3	135.9806	5.1669246607
June	4	136.0272	5.1708975373
June	5	136.0736	5.1748689006
June	6	136.1200	5.1788390741
June	7	136.1662	5.1828084082
June	8	136.2122	5.1867772826
June	9	136.2582	5.1907461105
June	10	136.3040	5.1947153428
June	11	136.3496	5.1986854723
June	12	136.3951	5.2026570402
June	13	136.4405	5.2066306418
June	14	136.4857	5.2106069354
June	15	136.5307	5.2145866506
June	16	136.5756	5.2185705999
June	17	136.6202	5.2225596924
June	18	136.6647	5.2265549493
June	19	136.7090	5.2305575243
June	20	136.7532	5.2345687280
June	21	136.7970	5.2385900582
June	22	136.8407	5.2426232385
June	23	136.8841	5.2466702671
June	24	136.9273	5.2507334797
June	25	136.9702	5.2548156324
June	26	137.0127	5.2589200110
June	27	137.0550	5.2630505798
June	28	137.0969	5.2672121872
June	29	137.1384	5.2714108557
June	30	137.1795	5.2756542053
July	1	137.2200	5.2799520895
July	2	137.2600	5.2843175880
July	3	137.2993	5.2887686308
July	4	137.3378	5.2933308160
July	5	137.3754	5.2980426654
July	6	137.4118	5.3029664212
July	7	137.4467	5.3082133835
July	8	137.4798	5.3140161793
July	9	137.5116	5.3210070441
July	10	137.5628	5.3312091210
July	11	137.6898	5.3405592121
July	12	137.8266	5.3466522674
July	13	137.9599	5.3516661563
July	14	138.0903	5.3561848203
July	15	138.2186	5.3604205657
July	16	138.3453	5.3644742164

Month	Date	λ (∘)	Distance (AU)
July	17	138.4707	5.3684017790
July	18	138.5949	5.3722377051
July	19	138.7183	5.3760047603
July	20	138.8409	5.3797188059
July	21	138.9628	5.3833913528
July	22	139.0841	5.3870310297
July	23	139.2048	5.3906444770
July	24	139.3250	5.3942369174
July	25	139.4448	5.3978125344
July	26	139.5641	5.4013747321
July	27	139.6831	5.4049263181
July	28	139.8016	5.4084696349
July	29	139.9198	5.4120066575
July	30	140.0377	5.4155390662
July	31	140.1553	5.4190683021
August	1	140.2725	5.4225956100
August	2	140.3895	5.4261220723
August	3	140.5062	5.4296486357
August	4	140.6225	5.4331761326
August	5	140.7387	5.4367052982
August	6	140.8546	5.4402367851
August	7	140.9702	5.4437711745
August	8	141.0856	5.4473089863
August	9	141.2007	5.4508506867
August	10	141.3157	5.4543966955
August	11	141.4303	5.4579473912
August	12	141.5448	5.4615031166
August	13	141.6591	5.4650641822
August	14	141.7731	5.4686308707
August	15	141.8869	5.4722034391
August	16	142.0006	5.4757821220
August	17	142.1140	5.4793671340
August	18	142.2272	5.4829586711
August	19	142.3402	5.4865569133
August	20	142.4530	5.4901620256
August	21	142.5657	5.4937741595
August	22	142.6781	5.4973934544
August	23	142.7904	5.5010200385
August	24	142.9024	5.5046540300
August	25	143.0143	5.5082955377
August	26	143.1260	5.5119446617
August	27	143.2375	5.5156014948
August	28	143.3488	5.5192661222
August	29	143.4599	5.5229386226
August	30	143.5709	5.5266190687
August	31	143.6817	5.5303075275